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Solitons and giants in matrix models

Ivan Andrić, Larisa Jonke and Danijel Jurman

Theoretical Physics Division, Rudjer Bošković Institute,
POB 180, 10002 Zagreb, Croatia
E-mail: iandric@irb.hr, larisa@irb.hr, djurman@irb.hr

Abstract: We present a method for solving BPS equations obtained in the collective-field approach to matrix models. The method enables us to find BPS solutions and quantum excitations around these solutions in the one-matrix model, and in general for the Calogero model. These semiclassical solutions correspond to giant gravitons described by matrix models obtained in the framework of AdS/CFT correspondence. The two-field model, associated with two types of giant gravitons, is investigated. In this duality-based matrix model we find the finite form of the n-soliton solution. The singular limit of this solution is examined and a realization of open-closed string duality is proposed.

Keywords: Field Theories in Lower Dimensions, Matrix Models, D-branes, Solitons Monopoles and Instantons.

Contents

1. Introduction

In the framework of AdS/CFT correspondence, the dynamics of giant gravitons was studied using matrix model with the harmonic-oscillator potential [1−3]. The interpretation of the matrix eigenvalues as fermions allows a description of gravitational excitations in the holographic dual of $N = 4$ SYM in terms of droplets in the phase space occupied by fermions. In particular, it was shown that giant gravitons expanding along AdS_5 and S^5 could be interpreted as a single excitation high above the Fermi sea, or as a hole in the Fermi sea, respectively [2]. In ref. [4] a correspondence between the general fermionic droplet and the classical ansatz for the AdS configuration was established. From ref. [1] it follows that giant gravitons are described by particular combinations of single-trace and multi-trace operators, known as Schur polynomials, pictorially represented by Young diagrams. For example, the afore-mentioned giants on AdS_5 and S^5 are represented by two types of Young tableaux: the one with the one-row diagram and the other with the one-column diagram [1, 2]. The matrix model with the harmonic-oscillator potential is related to the free matrix model via su(1,1) algebra which contains Hamitonians of both models as generators. As a consequence, their eigenstates are related via coherent states or by time reparametrization [5]. Therefore, a detailed discussion of BPS solutions of the matrix model without harmonic potential is relevant for giant graviton physics.

In this paper we analyse the model without the external potential and present a method for obtaining BPS solutions. Our starting point is the background-independent model defined by the action

$$S = \frac{1}{4}\int dt Tr\dot{M}^2(t), \tag{1.1}$$

where M is the $N \times N$ matrix. In ref. [6], the analysis of the states of the free, $\mathrm{U}(N)$ gauge-invariant matrix model represented by Young diagrams revealed two single-particle branches. The one-row diagram represents a single particle-like excitation above the Fermi sea with the dispersion law

$$\omega(k) = \frac{1}{2}(k + k_F)^2 - \frac{k_F^2}{2}, \tag{1.2}$$

and the one-column diagram represents a hole-like excitation in the Fermi sea with the dispersion law

$$\omega(k) = \frac{k_F^2}{2} - \frac{1}{2}(k - k_F)^2. \tag{1.3}$$

The dispersion laws (1.2) and (1.3) made possible the construction of effective collective-field Lagrangians for each of these branches [6], which are recognized as Lagrangians of $O(N)$ [7] and $Sp(N)$ [8] gauge-invariant matrix models. These matrix models are connected by duality [9], similarly to the duality between S^5 and AdS_5 giant gravitons [10]. As noted in ref. [11], the $O(N)$ and $Sp(N)$ gauge-invariant matrix models have their own string interpretation as unoriented (super)strings in two dimensions.

On a singlet subspace the free matrix model (1.1) reduces to the quantum mechanics of the N eigenvalues x_i of the matrix M. Introduction of the invariant measure over the matrix configuration space into the wavefunctions produces a prefactor $\prod_{i<j}^N (x_i - x_j)^\lambda$ which specifies generalized statistics [12], controlled by the parametar λ. For matrix models the parameter λ is related to the number of independent matrix degrees of freedom and takes the value $\lambda = 1/2, 1, 2$, in the case of $O(N)$, $\mathrm{U}(N)$ or $Sp(N)$ gauge-invariant matrices, respectively [9]. Finally, the dynamics of the eigenvalues is determined by the Hamiltonian of the Calogero type [13]

$$H_{\mathrm{CM}}(\{x\}; \lambda) = -\frac{1}{2}\sum_i \frac{d^2}{dx_i^2} + \lambda(\lambda - 1)\sum_{i<j} \frac{1}{(x_i - x_j)^2}. \tag{1.4}$$

Keeping in mind the relevance of the Calogero-type models for different branches of physics, for example in the quantum Hall effect [14], spin models [15], hydrodinamical models [16], systems of particles with generalized statistics [12], two-dimensional QCD [17] and black-hole physics [18], we use a general λ throughout the paper.

In order to study the large-N, continuum limit of the model defined by action (1.1), we use the collective-field approach, developed in ref. [19]. Introducing the collective fields ρ and π

$$\begin{aligned} &\rho(x,t) = \frac{1}{2}\int \frac{dk}{2\pi} e^{ikt} \rho_k(t), \;\; \rho_k(t) = \frac{1}{2} Tr e^{-ikM(t)}, \\ &[\partial_x \pi(x), \rho(y)] = -i\partial_x \delta(x - y), \end{aligned} \tag{1.5}$$

the matrix model (1.1) was recast in the following Hamiltonian:

$$\begin{aligned} H_{\mathrm{CM}}([\rho]; \lambda) = &\frac{1}{2}\int dx \rho(x)(\partial_x \pi(x))^2 + \frac{1}{8}\int dx \rho(x)\left[(\lambda - 1)\frac{\partial_x \rho(x)}{\rho(x)} - 2\pi\lambda\rho^H(x)\right]^2 - \\ &- \frac{\lambda}{2}\int dx\; \rho(x)\partial_x \frac{P}{x - y}\Big|_{y=x} - \frac{\lambda - 1}{4}\int dx\; \partial_x^2 \delta(x - y)\big|_{y=x}, \end{aligned} \tag{1.6}$$

where ρ^H is the Hilbert transform of ρ defined by

$$\rho^H(x) = -\frac{1}{\pi}⨍ dy \frac{\rho(y)}{x-y}.$$

The collective field $\rho(x)$ can be viewed as a bosonized 'fermionic' wave function of the aforementioned discrete case. Another approach to bosonization, suited for a finite number of fermions in one space dimension was proposed in ref. [20].

In attempt to go beyond the case of free fermions, the authors of ref. [21] start with two matrices and treat one of the matrices in the collective-field theory approach, while the other is treated in the coherent-state representation. This leads to the eigenvalue equations first found in [22], describing angular degrees of freedom of the single-matrix model. Another generalization of the free fermion picture was proposed in ref. [23], leading to the connection between multi-charge BPS operators of $N = 4$ SYM and the supersymmetric generalizations of the spin Calogero system. These developments motivated us to look also into the Lagrangian, introduced in refs. [24, 9], with two collective fields: one field associated with real symmetric matrix (with $O(N)$ invariance) and the other field associated with quaternionic matrix (with $Sp(N)$ invariance). This two-field model arises from the decomposition of the hermitian matrix into the sum of symmetric and antisymmetric matrix and can be thought of as a model of interacting giant gravitons.

The plan of the paper is the following. In section 2 we present a method for solving the BPS equation for the one-matrix model. The construction of the conformal field enables us to reduce the problem to the Riccati differential equation, which we relate to the Benjamin-Ono equation previously found in [6, 16]. Using the Riccati equation we investigate quantum excitations around the BPS solutions and related dispersion laws. In section 3 we analyse the two-field model and present the form of the BPS solution for n solitons. The singular limit of some of these solutions is examined and a realization of open/closed string duality is proposed. In the last section we summarize the main results and discuss some open questions.

2. One-matrix model

2.1 Riccati equation and boundary fields

Here we present a method for constructing BPS solutions of the Hamiltonian (1.6). The terms in the second line in (1.6) are singular counter terms, which do not contribute in the leading order in N. Assuming that the field $\partial_x \pi(x)$ vanishes, the leading part of the collective-field Hamiltonian (1.6) in the $1/N$ expansion is given by the effective potential

$$V_{\text{eff}} = \frac{1}{2}\int dx \rho(x) \left[\frac{\lambda-1}{2}\frac{\partial_x \rho(x)}{\rho(x)} - \lambda\pi\rho^H(x)\right]^2. \tag{2.1}$$

The effective potential can be rewritten as

$$V_{\text{eff}} = E_0 + \frac{1}{2}\int dx \rho(x) \left[\frac{\lambda-1}{2}\frac{\partial_x \rho(x)}{\rho(x)} + \frac{q(1-\lambda)}{2}\mathcal{P}\cot\left(\frac{qx}{2}+\varphi\right) - \lambda\pi\rho^H(x)\right]^2, \tag{2.2}$$

where the additional term in V_{eff} is defined as

$$\mathcal{P}\cot(qx/2+\varphi) = \lim_{\epsilon\to 0} \frac{\sin(qx+2\varphi)}{\cosh\epsilon - \cos(qx+2\varphi)}. \tag{2.3}$$

In (2.2), E_0 represents the terms which are to be subtracted because of the addition of $\mathcal{P}\cot(qx/2+\varphi)$ term into the square brackets:

$$\begin{aligned} E_0 = &\frac{q(\lambda-1)^2}{4}\int dx\partial_x\rho(x)\mathcal{P}\cot\left(\frac{qx}{2}+\varphi\right) - \frac{q^2(\lambda-1)^2}{8}\int dx\rho(x)\mathcal{P}\cot^2\left(\frac{qx}{2}+\varphi\right) - \\ &-\frac{q\pi\lambda(\lambda-1)}{2}\int dx\rho(x)\rho^H(x)\mathcal{P}\cot\left(\frac{qx}{2}+\varphi\right). \end{aligned} \tag{2.4}$$

Assuming the compact support $[-L/2, L/2]$, using the normalization condition $\int dx\rho(x) = N$ and the identity

$$(f^H g + f g^H)^H = f^H g^H - fg + f_0 g_0, \quad \begin{pmatrix} f_0 \\ g_0 \end{pmatrix} = \frac{1}{L}\int dx \begin{pmatrix} f(x) \\ g(x) \end{pmatrix}, \tag{2.5}$$

and performing partial integration one obtains

$$\begin{aligned} E_0 = &\frac{qN(1-\lambda)}{8}\left[(1-\lambda)q + 2\pi\lambda\frac{N}{L}\right] + \frac{q(\lambda-1)^2}{4}\rho(x)\mathcal{P}\cot\left(\frac{qx}{2}+\varphi\right)\Big|_{-L/2}^{L/2} + \\ &+\frac{q\pi\lambda(\lambda-1)L}{4}\left(\rho^2(x) - {\rho^H}^2(x)\right)\Big|_{x=-2\varphi/q}. \end{aligned} \tag{2.6}$$

At this point, q and φ are free parameters to be determined by boundary conditions such that the last two terms in (2.6) should vanish and by the condition that E_0 should be a non-negative constant. The precise choice of q and φ satisfying these conditions determines different solutions of the model and is discussed in the next subsection. Here we proceed with the analysis of the effective potential (2.2). With E_0 being a constant, the contribution of V_{eff} to the Hamiltonian is minimized by a solution of the integro-differential Bogomol'nyi-type equation

$$\partial_x\rho = q\mathcal{P}\cot\left(\frac{qx}{2}+\varphi\right)\rho + \frac{\lambda\pi}{\lambda-1}2\rho\rho^H. \tag{2.7}$$

Taking the Hilbert transform of eq. (2.7), we find the equation for ρ^H,

$$\partial_x\rho^H = q\mathcal{P}\cot\left(\frac{qx}{2}+\varphi\right)\rho^H - q\rho_0 - \frac{\lambda\pi}{\lambda-1}\left(\rho^2 - {\rho^H}^2 - \rho_0^2\right). \tag{2.8}$$

We construct the conformal field Φ containing only the positive frequency part of ρ

$$\Phi = \rho^H + i\rho = \frac{1}{\pi}\int dz \frac{\rho(z)}{z-x-i\epsilon}, \tag{2.9}$$

and satisfying the Riccati differential equation

$$\partial_x\Phi = \frac{\lambda\pi}{\lambda-1}\Phi^2 + q\mathcal{P}\cot\left(\frac{qx}{2}+\varphi\right)\Phi + \frac{\lambda\pi\rho_0^2}{\lambda-1} - q\rho_0. \tag{2.10}$$

The physical interpretation of the Riccati differential equation can be found from the relation to the Benjamin-Ono equation [16]. Taking into account the Bogomol'nyi limit, we evaluate the fields u^+ and u^- from ref. [16]:

$$u^- = \sqrt{\lambda}\pi\Phi,\ u^+ = \frac{\lambda-1}{\sqrt{\lambda}}\frac{q}{2}\mathcal{P}\mathrm{cot}\left(\frac{qx}{2}+\varphi\right) \tag{2.11}$$

and then plugging (2.11) into the Benjamin-Ono equation, we obtain eq. (2.10).

Obviously, if the field ρ is a solution of eq. (2.7), then the field Φ necessarily satisfies eq. (2.10). The converse is not true in general, and to obtain the field ρ from the solution of the Riccati equation (2.10), it is sufficient that the condition

$$\Phi^H(x) = i\Phi(x) + \rho_0 \tag{2.12}$$

holds. In this case, the solution of eq. (2.7) is simply given as $\rho = -i(\Phi - \Phi^*)/2$. Equation (2.10) can be further transformed into the second-order Schrödinger-like differential equation by making the substitution $\Phi = (1-\lambda)\partial_x v/(\lambda\pi v)$:

$$\partial_x^2 v = q\mathcal{P}\mathrm{cot}\left(\frac{qx}{2}+\varphi\right)\partial_x v - \frac{\lambda\pi\rho_0}{1-\lambda}\left(\frac{\lambda\pi\rho_0}{1-\lambda}+q\right)v. \tag{2.13}$$

2.2 Semiclassical solutions

The construction of the Riccati equation enables us to obtain the semiclassical static solutions. Generally, there are two methods for solving the Riccati equation. One method is to construct a general solution from the known particular solution, while the other method is based on solving the second-order linear differential equation (2.13). Applying both methods, we find some interesting solutions, given in table 1.

In the following analysis we differentiate two possibilities: $\lambda < 1$ and $\lambda > 1$. First, we discuss the case $\lambda < 1$. The particular solution of the Riccati equation (2.10) for non-vanishing ρ_0 is given in the first row of table 1. The parameter of this solution satisfies the relation

$$e^t = 1 + \frac{q(1-\lambda)}{\lambda\pi\rho_0}. \tag{2.14}$$

Taking into account the boundary conditions

$$\rho^H(-\frac{2\varphi}{q}) = \rho(-\frac{2\varphi}{q}) = 0,\ \mathcal{P}\mathrm{cot}(\frac{qL}{4}+\varphi) = 0, \tag{2.15}$$

we find

$$q = 2\pi M/L,\ M \in \mathbb{N}, \tag{2.16}$$

where the number M can be interpreted as the number of solitons. In order to have odd M, we take $\varphi = 0$, whereas for even M we take $\varphi = \pi/2$. Taking into account the normalization condition, we find

$$e^t = 1 + \frac{2M(1-\lambda)}{N\lambda}. \tag{2.17}$$

From the M-soliton solution in the limit $L \to \infty$, keeping ρ_0 fixed and defining

$$b = (1-\lambda)/(\lambda\pi\rho_0), \tag{2.18}$$

λ	$\Phi_s(x)$	$\rho_s(x)$	E_0
$\lambda < 1$	$\frac{iq(1-\lambda)}{\lambda\pi(e^t-1)}\frac{1-e^{i(qx+2\varphi)}}{1-e^{-t}e^{i(qx+2\varphi)}}$	$\frac{q(1-\lambda)\coth(t/2)}{2\pi\lambda}\frac{1-\cos(qx+2\varphi)}{\cosh t-\cos(qx+2\varphi)}$	$\frac{(1-\lambda)\pi^2}{2L^2}[\lambda N^2M+(1-\lambda)NM^2]$
	$\frac{1-\lambda}{\lambda\pi b}\frac{ix}{x+ib}$	$\frac{1-\lambda}{\lambda\pi b}\frac{x^2}{x^2+b^2}$	$\frac{(1-\lambda)^3}{2\lambda b^2}$
$\lambda > 1$	$\frac{ik(\lambda-1)}{2\pi\lambda}\frac{1+e^{-t}e^{ikx}}{1-e^{-t}e^{ikx}}$	$\frac{k(\lambda-1)}{2\pi\lambda}\frac{\sinh t}{\cosh t-\cos kx}$	0
	$\frac{1-\lambda}{\lambda\pi}\frac{1}{x+ib}$	$\frac{\lambda-1}{\lambda\pi}\frac{b}{x^2+b^2}$	0
$\lambda \lesseqgtr 1$	$i\rho_0$	ρ_0	0

Table 1: BPS solutions

we find the one-soliton solution ($M = 1$, $\varphi = 0$) obtained in refs. [25, 26]. The energy is just the energy of one soliton obtained by taking the corresponding limit. The uniform zero-energy solution $\rho(x) = \rho_0$ is obtained in the limit $q \to 0$, taking $\varphi = \pi/2$.

Next we discuss the case $\lambda > 1$. We take $q = 0$, $\varphi = \pi/2$, thus eliminating the term $\mathcal{P}$cot from eq. (2.10), and obtain the general solution of the Riccati equation (2.10) for

$$\rho_0 = \frac{(\lambda-1)k}{2\pi\lambda},\; k = 2\pi M/L. \tag{2.19}$$

It is given in the third row of table 1, where t is a non-negative free parameter. An additional solution could be obtained from the general solution in the limit $L \to \infty$, taking $t = 2\pi b/L$. In the case $t \to \infty$, we obtain the constant density solution $\rho(x) = \rho_0$. Taking into account the normalization condition we obtain that the number of solitons M exceeds the number of particles N giving us the relation

$$\lambda = M/(M-N). \tag{2.20}$$

Solitons on the compact support from table 1 are of the same shape as solitons in the Sutherland model [27], thus reflecting the fact that the two models are interrelated via the periodicity condition. Using the condensed-matter language, we interpret the M-soliton solutions as soliton trains. In the large-M limit, these solutions can be viewed as crystal-like structures with periodicity $2\pi/q$ or $2\pi/k$. The one-soliton solutions are regarded as composite particles (quasi-particles). Resembling the holes (lumps), these one-soliton solutions enable us to identify the M-soliton solutions found for $\lambda < 1$ ($\lambda > 1$) as dark (bright) solitons. In ref. [26] it was shown that adding the term $(1-\lambda)/(x-z)$ into the effective potential was equivalent to the extraction of the prefactor $\prod_i(x_i - z)^{1-\lambda}$ from the wave function of the Hamiltonian (1.4). This equivalence enables us to associate a quasi-particle located at z with the prefactor of the wave function. Consequently, the additional term $\mathcal{P}\cot(qx/2 + \varphi)$ is associated with the prefactor describing M equidistant quasi-particles.

The soliton solutions we have found in the collective-field formulation of the free matrix model correspond to the particle and hole states in the system (1.4) of nonrelativistic fermions [6]. Owing to the $su(1,1)$ dynamical symmetry [9], the eigenstates of the Hamiltonian (1.4) can be represented as generalized coherent states of the same Hamiltonian with

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the additional harmonic potential interaction between fermions [28–30]. The particle and hole states in the system of fermions with the harmonic potential interaction correspond to the giant gravitons of a 1/2 BPS sector of $N = 4$ SYM [1–4]. Therefore, our solutions correspond to the coherent states of the matrix model with the harmonic potential, i.e. to the quasi-classical CFT duals of the giant gravitons in AdS constructed in ref. [3]. The nonexistence of the quasi-classical CFT dual of the single giant graviton on the sphere S^5 is reflected throught the relation (2.20), from which it follows that the soliton with $M = 1$ in the $\lambda > 1$ case is non-normalizable since M must exceed N, in accordance with the conclusion of ref. [3].

At this point we would like to emphasize a simple relation between systems with $\lambda < 1$ and those with $\lambda > 1$. By substituting $\lambda\rho(x) = \alpha - m(x)$ into eq. (2.7) for $\lambda > 1$ (without the term $\mathcal{P}$cot) and by inserting explicit forms of the solutions for the term ρ^H/ρ, we find that the field m satisfies eq. (2.7) for $\lambda' = 1/\lambda < 1$ (with the term $\mathcal{P}$cot). This agrees with the result obtained in ref. [31] in the k-space ($\rho_k \to -m_k/\lambda$). The difference is that our relation is valid for the BPS equations, whereas in ref. [31] the duality relation connects quantum Hamiltonians. Recall that the solution of the form similar to that given in the third row of table 1 was found as a solution of the dynamic equations of motion of the Calogero model in ref. [25]. This signalizes that there is a generalization of our method for dynamic equations of motion (some progress has been made in ref. [16]). Finally, we stress that the construction of the Riccati equation is possible for the model trapped in the harmonic well, providing a new method for analysing this model.

2.3 Quantum excitations around semiclassical solutions

To get an insight into the dynamics of quantum excitations, we expand the Hamiltonian (1.6) around the semiclassical solution

$$\rho(x,t) = \rho_s(x) + \partial_x \eta(x,t), \tag{2.21}$$

where η is a small density quantum fluctuation around the soliton solution ρ_s of eq. (2.7). The quadratic part of the Hamiltonian can be written in the following form:

$$H^{(2)} = \frac{1}{2}\int dx \rho_s(x) A^\dagger(x) A(x), \tag{2.22}$$

where we have introduced the operators A

$$A = -\pi_\eta + i\left[\frac{(\lambda-1)}{2}\partial_x \frac{\partial_x \eta}{\rho_s} - \pi\lambda\partial_x \eta^H\right], \tag{2.23}$$

satisfying the following equal-time commutation relation:

$$\left[A(x), A^\dagger(y)\right] = (1-\lambda)\partial^2_{xy}\frac{\delta(x-y)}{\rho_s(x)} + 2\lambda\partial_x \frac{P}{x-y}. \tag{2.24}$$

Using the equation of motion $\dot{A}(x,t) = i[H, A(x,t)]$, we obtain the equation

$$\left[-i\partial_t + \frac{\lambda-1}{2}\frac{\partial_x \rho_s}{\rho_s}\partial_x - \frac{\lambda-1}{2}\partial_x^2\right](\rho_s A) = -\lambda\pi\rho_s\partial_x(\rho_s A)^H. \tag{2.25}$$

Taking the Hilbert transform of this equation, and using eq. (2.7), we find

$$\left[-i\partial_t + \frac{\lambda-1}{2}\frac{\partial_x\rho_s}{\rho_s}\partial_x - \frac{\lambda-1}{2}\partial_x^2\right](\rho_s A)^H = \lambda\pi\rho_s\partial_x(\rho_s A). \tag{2.26}$$

Defining the fields

$$\Phi_s^\pm = \rho_s^H \pm i\rho_s,\ \phi^\pm = (\rho_s A)^H \pm i(\rho_s A), \tag{2.27}$$

we find that $\phi^\pm$ satisfies

$$\left\{i\partial_t - \left[\lambda\pi\Phi_s^\pm + \frac{q(\lambda-1)}{2}\mathcal{P}\cot\left(\frac{qx}{2}+\varphi\right)\right]\partial_x + \frac{\lambda-1}{2}\partial_x^2\right\}\phi^\pm = 0. \tag{2.28}$$

This equation can be obtained from the Riccati equation (2.10) by adding $2i\partial_t\phi^\pm/(\lambda-1)$ on the l.h.s. and expanding $\Phi(x,t)$ around the solution $\Phi_s(x)$, $\Phi^\pm(x,t) = \Phi_s^\pm(x) + \partial_x\phi^\pm(x,t)$, keeping only terms linear in ϕ. Therefore, one can interpret the field ϕ as a fluctuation around the conformal field Φ_s. Solving eq. (2.28) for the solutions found in section 3, we obtain the following results:

- the operator A is given by

$$A = \frac{2\pi}{L}\sum_{n,s} e^{i\omega_n t} f_{n,s}(x)\left[\theta(\omega_n)a_{n,s} + \theta(-\omega_n)a_{n,s}^\dagger\right], \tag{2.29}$$

  where the operators $a_{n,s}$ satisfy

$$[a_{n,s}, a_{m,s'}^\dagger] = |\omega_n| L/\pi\delta_{nm}\delta_{ss'} \tag{2.30}$$

  and the functions $f_{n,s}$ are orthonormalized with respect to the measure $\rho_s(x)$ as follows:

$$\int_{-L/2}^{L/2} dx\rho_s(x) f_{n,s}^*(x) f_{m,s'}(x) = \frac{L}{2\pi}\delta_{nm}\delta_{s,s'}; \tag{2.31}$$

- the Hamiltonian up to quadratic terms is given by

$$H = E_0 + \frac{\pi}{L}\sum_{n,s} a_{n,s}^\dagger a_{n,s} + \sum_{n,s}\theta(-\omega_n)|\omega_n|. \tag{2.32}$$

  The functions $f_{n,s}$ and the eigenvalues ω_n for all solutions found in section 3 are given in table 2, where $k_0 = |\lambda\pi\rho_0/(\lambda-1)|$.

Comparing the known dispersion laws (1.2), (1.3) for the quantum excitations on the uniform background, obtained from the Young diagrams in ref. [6] with the dispersion laws given in table 2 we find an agreement. At the semiclassical level we obtain that the system can be in the fluid phase (uniform density) as well as in the crystal-like configuration. So, it would be interesting to calculate the phase transition amplitudes and also to calculate the correlation functions determined in this approach by the quadratic Hamiltonian.

λ	ρ_s	$f_{n,\pm}$		ω_n
$\lambda<1$	$\frac{q(1-\lambda)\coth(t/2)}{2\pi\lambda}\frac{1-\cos(qx+2\varphi)}{\cosh t-\cos(qx+2\varphi)}$	$\sqrt{\frac{\lambda(k_0+q)(k_n+q)}{4(1-\lambda)k_0k_n(2k_0+q)}}\left(1-\frac{k_ne^{\pm i(qx+2\varphi)}}{k_n+q}\right)\left(1-\frac{k_0e^{\mp i(qx+2\varphi)}}{k_0+q}\right)\frac{e^{\pm i(k_n-k_0)x}}{1-\cos(qx+2\varphi)}$	$k_n>k_0$	$\frac{1-\lambda}{2}(k_n+k_0+q)(k_n-k_0)$
	$\frac{1-\lambda}{\lambda\pi b}\frac{x^2}{x^2+b^2}$	$\sqrt{\frac{\lambda}{2k_0(1-\lambda)}}\left(1\pm\frac{i}{k_nx}\right)\left(1\mp\frac{i}{k_0x}\right)e^{\pm i(k_n-k_0)x}$	$k_n>k_0$	$\frac{1-\lambda}{2}\left(k_n^2-k_0^2\right)$
	ρ_0	$\frac{1}{\sqrt{2\pi\rho_0}}e^{\pm i(k_n-k_0)x}$	$k_n>k_0$	$\frac{1-\lambda}{2}\left(k_n^2-k_0^2\right)$
$\lambda>1$	$\frac{k(\lambda-1)}{2\pi\lambda}\frac{\sinh t}{\cosh t-\cos kx}$	$\sqrt{\frac{\lambda}{2k_0(\lambda-1)\left(1-e^{-2t}\right)}}\left(1-e^{-t}\,e^{\mp 2ik_0x}\right)e^{\pm i(k_n+k_0)x}$	$k_n>-k_0$	$\frac{\lambda-1}{2}\left(k_0^2-k_n^2\right)$
	$\frac{\lambda-1}{\lambda\pi}\frac{b}{x^2+b^2}$	$\sqrt{\frac{\lambda}{2b(\lambda-1)}}(x\mp ib)\,e^{\pm ik_nx}$	$k_n>0$	$-\frac{\lambda-1}{2}k_n^2$
	ρ_0	$\frac{1}{\sqrt{2\pi\rho_0}}e^{\pm i(k_n+k_0)x}$	$k_n>-k_0$	$\frac{\lambda-1}{2}\left(k_0^2-k_n^2\right)$

Table 2: Excitations around BPS solutions

3. Duality-based matrix model

3.1 Semiclassical solutions

In attempt to go beyond the case of free fermions, in this section we analyse a generalization of the hermitian matrix model, introduced in refs. [24, 9]. Actually, this model was first formulated in ref. [32] as a duality-based generalization of the Calogero model, and is defined by the Hamiltonian

$$H(x,z) = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{1}{2}\sum_{i\neq j}^{N} \frac{\lambda(\lambda-1)}{(x_i-x_j)^2} + \frac{1}{2}\sum_{i,\alpha}^{N,M} \frac{(\kappa+\lambda)(\kappa-1)}{(x_i-Z_\alpha)^2} + \\ + \frac{\lambda}{\kappa}\left[\sum_{\alpha=1}^{M} \frac{p_\alpha^2}{2} + \frac{1}{2}\sum_{\alpha\neq\beta}^{M} \frac{\kappa^2/\lambda\left(\kappa^2/\lambda-1\right)}{(Z_\alpha-Z_\beta)^2}\right]. \tag{3.1}$$

In ref. [9], it was shown that for $\lambda = 1/2$ this model arises from the decomposition of the hermitian matrix into the sum of symmetric and antisymmetric matrix. Transformation into the hydrodynamic formulation, in terms of the density operators $\rho(x)$, $m(x)$ and the corresponding conjugate operators for $\kappa = 1$, results in the hermitian collective-field Hamiltonian [9]

$$H = \frac{1}{2}\int dx\rho(x)(\partial_x\pi_\rho(x))^2 + \frac{\lambda}{2}\int dx m(x)(\partial_x\pi_m(x))^2 + \\ + \int dx\frac{\rho(x)}{2}\left[\frac{\lambda-1}{2}\frac{\partial_x\rho(x)}{\rho(x)} + \fint dy\frac{\lambda\rho(y)}{x-y} + \fint dy\frac{m(y)}{x-y}\right]^2 + \\ + \int dx\frac{m(x)}{2\lambda}\left[\frac{1-\lambda}{2}\frac{\partial_x m(x)}{m(x)} + \fint dy\frac{m(y)}{x-y} + \fint dy\frac{\lambda\rho(y)}{x-y}\right]^2 - \\ - \frac{\lambda}{2}\int dx\rho(x)\partial_x\frac{P}{x-y}\Big|_{x=y} - \frac{1}{2}\int dx m(x)\partial_x\frac{P}{x-y}\Big|_{x=y}. \tag{3.2}$$

The terms in the last line in (3.2) are singular counter terms, which do not contribute in the leading order in N and M. This model was analysed in ref. [33], where some solutions were constructed. Here, we use the Riccati equation to construct some new solutions in this duality-based model. We are looking for the solutions of two coupled Bogomol'nyi equations,

$$(\lambda-1)\partial_x\rho - 2\pi\rho(\lambda\rho^H + m^H) = 0, \tag{3.3}$$

$$(1-\lambda)\partial_x m - 2\pi m(\lambda\rho^H + m^H) = 0. \tag{3.4}$$

Based on the duality, we make an ansatz $m^H = -\lambda\alpha\rho^H/\rho$. Equation (3.3) now becomes

$$(\lambda-1)\partial_x\rho - 2\lambda\pi\rho\rho^H + 2\lambda\alpha\pi\rho^H = 0. \tag{3.5}$$

Following the method from section 2, we construct the field $\Phi = \rho^H + i\rho$ which satisfies the following Riccati equation:

$$\partial_x\Phi = \frac{\lambda\pi}{\lambda-1}\Phi^2 - i\frac{2\lambda\pi\alpha}{\lambda-1}\Phi + \frac{\lambda\pi\rho_0}{\lambda-1}(\rho_0 - 2\alpha). \tag{3.6}$$

The general solution of this equation constructed from the constant solution $\Phi = i\rho_0$ is

$$\Phi(x) = i\rho_0 - \frac{\lambda - 1}{\lambda\pi}\frac{iqce^{iqx}}{1 + ce^{iqx}}, \; q = \frac{2\lambda\pi(\alpha - \rho_0)}{1 - \lambda} > 0. \tag{3.7}$$

The solutions for ρ and m ($c = e^{i\phi - u - v}$, $|c| < 1$) are

$$\begin{aligned} \rho(x) &= \alpha\frac{\cosh(u - v) + \cos(qx + \phi)}{\cosh(u + v) + \cos(qx + \phi)}, \; m(x) = \frac{\tilde{c}}{\rho(x)}, \\ q &= \frac{4\lambda\pi\alpha}{1 - \lambda}\frac{\sinh u \sinh v}{\sinh(u + v)}, \; \frac{\tilde{c}}{\lambda\alpha^2} = \frac{\sinh(u - v)}{\sinh(u + v)}, \; u > v > 0. \end{aligned} \tag{3.8}$$

Taking into account the normalization conditions and the compact support $[-L/2, L/2]$, $q = 2\pi n/L$, we find the following relations:

$$\begin{aligned} \rho_0 &= \frac{N}{L}, \; \alpha = \frac{\lambda N + (1 - \lambda)n}{\lambda L}, \; \coth u = 2 + \frac{\lambda N - M}{(1 - \lambda)n}, \\ m_0 &= \frac{M}{L}, \; \frac{\tilde{c}}{\alpha} = \frac{M - (1 - \lambda)n}{L}, \; \coth v = \frac{\lambda N + M}{(1 - \lambda)n}. \end{aligned} \tag{3.9}$$

From the solution (3.8) taking $\phi = \pi$, $\sinh(u/2 - v/2) = aq/2$, $\sinh(u/2 + v/2) = bq/2$, $b > 0$, and taking the limit $q \to 0$, we obtain the one-soliton solution

$$\rho(x) = \alpha\frac{x^2 + a^2}{x^2 + b^2}, \; m(x) = \frac{\lambda\alpha^2 a}{b\rho(x)}, \; a^2 = b^2 + \frac{\lambda - 1}{\lambda\pi\alpha}b. \tag{3.10}$$

Here we would like to emphasize that the properties of the solutions of the dual model (3.2) resemble the properties of the dual giant gravitons on AdS ($\lambda < 1$) and on the sphere ($\lambda > 1$) from refs. [10, 1, 2].

3.2 Semiclassical solutions in the singular limit

In this subsection we discuss the existence of the singular solutions of the duality-based model and the methods for finding them. Inspired by the one-soliton solution of the model introduced in [24], we proposed the existence of the multi-soliton solutions in the same paper. In the paper [33] we discussed these solutions in the context of the new matrix-model interpretation of the duality-based Hamiltonian. We started from an assumption that there exists the finite form of a multi-soliton solution. In the case of the one-soliton solution, the finite form of the solution was known and we noticed that, in the singular limit, the conditions on the parameters of the solution were reduced. This indicates that in the singular limit, the Bogomol'nyi equations are less sensitive to the details of the precise form of the finite solution. In the hope of getting a hint about the finite form of a multi-soliton solution, we used a simple ansatz which in itself was not a solution in its finite form, but had the same singular limit as the finite solution. We found that the ansatz solved the equations in the singular limit. Although the calculation of the contribution to the Hamiltonian leads to an ambiguity in order of taking limits, this problem can be avoided by introduction of the finite support. Now, we would like to confirm the existence of singular solutions and the correctness of the calculations performed in the paper [33] by using the finite form of the solution constructed in subsection 3.1. Taking the limit $u-v=2\epsilon\to 0$ of the solution (3.8), we find

$$\rho(x)=\alpha\frac{\cos^2(\frac{qx+\phi}{2})}{\sinh^2 v+\cos^2(\frac{qx+\phi}{2})},\ \alpha=\frac{(1-\lambda)q}{2\lambda\pi}\coth v,$$
$$m(x)=(1-\lambda)\sum_{i=-\infty}^{\infty}\delta(x-x_i),\ x_i=\frac{(2i+1)\pi-\phi}{q}.$$

Using the product representation for the trigonometric function we find that the ∞-soliton solution exactly matches the form proposed in our paper [33]. Furthermore, by taking the compact support of lenght L, we find the finite number of solitons. In the case that q is small, we find that this solution reduces to the rational ansatz, as proposed in papers [24, 33].

In the comment made by V. Bardek and S. Meljanac in [34] regarding this part of our published work [33], the authors claim that such solutions do not exist. The authors based their claims on the symbolic manipulation with delta functions assuming the independece of the regularization procedure. As we are aware that solving nonlinear coupled equations is sensitive to the regularization used, the application of the symbolic manipulation is not suitable and does not give a unique answer because the product of distributions is not uniquely defined. The authors also claim that their paper [35] gives convicing arguments that a singular form of the solution does not exist, but they simply have not checked the limit $c\to 0$ in that paper.

3.3 Open-closed string duality?

Solutions of the equations obtained in the collective-field formulation of matrix models are interpretated as giant gravitons. Although the existence of multi-soliton solutions in their finite form is also an interesting question from the point of view of integrable systems, the limiting form when one of the fields behaves as a sum of delta functions deserves special attention from the point of view of matrix theory. Namely, in this singular configuration, it is reasonable to describe the $m(x)$ part of the Hamiltonian by a discrete set of variables. Furthermore, we can interpret the continuum part of the Hamiltonian as closed string theory and the discrete part of the Hamiltonian as open string theory [2]. So, by studying the relation between these two Hamiltonians we might get a better insight into the open-closed string duality.

The matrix model (1.1) (the Calogero model) possesses the symmetry of the action generated by the operators closing the $su(1,1)$ algebra [32, 9]:

$$[T_+,T_-]=-2T_0,\ [T_0,T_\pm]=\pm T_\pm,$$

The generators of the algebra in the discrete case are

$$T_+(\{x\};\lambda)=\frac{1}{2}\sum_{i=1}^N\partial_i^2+\frac{\lambda}{2}\sum_{i\neq j}^N\frac{1}{x_i-x_j}(\partial_i-\partial_j),$$
$$T_0(\{x\};\lambda)=-\frac{1}{2}\left(\sum_{i=1}^N x_i\partial_i+E_0\right),\ T_-(\{x\};\lambda)=\frac{1}{2}\sum_{i=1}^N x_i^2, \tag{3.11}$$

and in the continuous, large-N case,

$$
\begin{aligned}
T_+([\rho];\lambda) &= -\frac{1}{2}\int dx\rho(x)(\partial_x\pi(x))^2+\frac{1}{2}\int dx\left((\lambda-1)\partial_x\rho(x)+2\lambda\rho(x)\fint dy\frac{\rho(y)}{x-y}\right)\partial_x\pi(x),\\
T_0([\rho];\lambda) &= -\frac{1}{2}\left(i\int dx x\rho(x)\partial_x\pi(x)+E_0\right),\ T_-([\rho];\lambda)=\frac{1}{2}\int dx x^2\rho(x),
\end{aligned}
\tag{3.12}
$$

where the ground-state energy is $E_0(N,\lambda)=(\lambda N(N-1)+N)/2$.

Furthermore, it was shown in [32, 24, 36, 9] that there existed a strong-weak coupling duality in the Calogero model and here we briefly review the main results. In the following we use the abbreviation $[\bullet]$ for the arguments of operators, depending on which case is under consideration (discrete or continuous) and analogously for the dual system $[\circ]$. We also need the following definitions:

$$
\ln J([\bullet],\lambda)=\begin{cases}\lambda\sum_{i\neq j}\ln(x_i-x_j) & \text{discrete}\\ (\lambda-1)\int dx\rho(x)\ln\rho(x)+\lambda\int\int dxdy\rho(x)\ln|x-y|\rho(y) & \text{continuous,}\end{cases}
\tag{3.13}
$$

$$
\ln V([\bullet],[\circ])=\begin{cases}\sum_{i,\alpha}\ln(x_i-z_\alpha) & \text{discrete}-\text{discrete}\\ \lim_{\varepsilon\to 0}\int\int dxdz\rho(x)\ln(x-z-i\varepsilon)m(z) & \text{continuous}-\text{continuous}\\ \lim_{\varepsilon\to 0}\sum_\alpha\int dx\rho(x)\ln(x-z_\alpha-i\varepsilon) & \text{continuous}-\text{discrete.}\end{cases}
\tag{3.14}
$$

The strong-weak duality is displayed as follows:

$$
\begin{aligned}
T_0([\bullet],\lambda)V([\bullet],[\circ]) &= \left\{-T_0([\circ],1/\lambda)-\frac{1}{2}[NM+E_0(N,\lambda)+E_0(M,1/\lambda)]\right\}V([\bullet],[\circ]),\\
T_+([\bullet],\lambda)V([\bullet],[\circ]) &= -\lambda T_+([\circ],1/\lambda)V([\bullet],[\circ]).
\end{aligned}
\tag{3.15}
$$

The operators $T_+([\bullet],\lambda)$ are equivalent to the Hamiltonians (1.4), (1.6)

$$
H_{\mathrm{CM}}([\bullet],\lambda)=-J^{\frac{1}{2}}([\bullet],\lambda)T_+([\bullet];\lambda)J^{-\frac{1}{2}}([\bullet],\lambda)
\tag{3.16}
$$

while $T_0([\bullet],\lambda)$ are equivalent to the same Hamiltonians with an additional harmonic-well potential (Calogero-Sutherland Hamiltonians):

$$
\begin{aligned}
H_{\mathrm{CS}}([\bullet];\lambda,\omega) &= -2\omega J^{\frac{1}{2}}([\bullet],\lambda)S([\bullet];\lambda,\omega)T_0([\bullet];\lambda)S^{-1}([\bullet];\lambda,\omega)J^{-\frac{1}{2}}([\bullet],\lambda)=\\
&= H_{\mathrm{CM}}([\bullet],\lambda)+\omega^2T_-([\bullet];\lambda),
\end{aligned}
\tag{3.17}
$$

where

$$
S([\bullet];\lambda,\omega)=e^{-\omega T_-([\bullet];\lambda)}e^{-\frac{1}{2\omega}T_+([\bullet];\lambda)}.
\tag{3.18}
$$

Consequently, we interpret

$$
H_{\mathrm{CM}}([\circ];1/\lambda)\equiv-J^{\frac{1}{2}}([\circ];1/\lambda)T_+([\circ];1/\lambda)J^{-\frac{1}{2}}([\circ];1/\lambda)
\tag{3.19}
$$

and

$$
\begin{aligned}
H_{CS}([\circ];1/\lambda,\omega/\lambda)&\equiv-\frac{2\omega}{\lambda}J^{\frac{1}{2}}([\circ],1/\lambda)S([\circ];1/\lambda,\omega/\lambda)T_0([\circ];1/\lambda)S^{-1}([\circ];1/\lambda,\omega/\lambda)J^{-\frac{1}{2}}([\circ],1/\lambda)=\\
&= H_{\mathrm{CM}}([\circ],1/\lambda)+\frac{\omega^2}{\lambda^2}T_-([\circ];1/\lambda),
\end{aligned}
\tag{3.20}
$$

as Hamiltonians of the dual system. In the papers [32, 24, 36] the duality relations were used for construction of the spectrum generating algebra for the two-family model, while here we show that they can be used for construction of the eigenstate of one system from the known eigenstate of the dual system. Suppose that we know the eigenstate $\psi_{\{E\}}$ with energy E, satisfying

$$H_{\rm CS}([\bullet];\lambda,\omega)\psi_{\{E\}} = E\psi_{\{E\}}. \tag{3.21}$$

Then, the eigenstate $\phi_{\{\tilde{E}\}}$ of the dual system, with energy $\tilde{E}$,

$$H_{\rm CS}\left([\circ];1/\lambda,\omega/\lambda\right)\phi_{\{\tilde{E}\}} = \tilde{E}\phi_{\{\tilde{E}\}}, \tag{3.22}$$

is given by

$$\phi_{\{\tilde{E}\}} = J^{\frac{1}{2}}\left([\circ],1/\lambda\right)e^{-\frac{\omega}{\lambda}T_-([\circ];1/\lambda)}\int_{\mathcal{V}}\psi_{\{E\}}J^{\frac{1}{2}}([\bullet],\lambda)e^{-\omega T_-([\bullet];\lambda)}V([\bullet],[\circ]), \tag{3.23}$$

where the integration is performed over the corresponding configuration space and

$$\lambda\tilde{E} + E = \omega[NM + E_0(N,\lambda) + E_0(M,1/\lambda)]. \tag{3.24}$$

The corresponding dual relation also holds for the Hamiltonians $H_{\rm CM}$ because the eigenstates of these are realized as coherent states of the $H_{\rm CS}$ [28 – 30], or in another approach by the use of unconventional separation of variables in the Schrödinger problem [37]. Now, the quantum mechanics (1.4) of the eigenvalues x_i of M was regarded as an open string/D-brane description of the corresponding string theory [38]. On the other hand, the collective field defined in the large-N limit of the matrix model (3.19) represents the closed string excitations [39]. Inspired by this interpretation we propose an explicit realization of open-closed string duality. The relation (3.23) tells us how to construct a wave functional of closed string excitations described by the collective field ρ from the wave function of dual M open string excitations, and vice versa.

4. Conclusion

Motivated by the relevance of soliton solutions in matrix models for giant graviton physics, we have addressed the problem of existence of these solutions and have analysed their properties. We have introduced a powerful method for obtaining these BPS solutions in the collective-field approach. The method is based on the construction of a boundary conformal field out of the density of eigenvalues of a matrix, satisfying the Riccati differential equation. This method extends to the related Calogero models, in particular to the model with the harmonic-well potential. Furthermore, we have established the relation between the hydrodynamic Benjamin-Ono equation and the Riccati equation, suggesting that our method could be extended to non-BPS equations by inclusion of dynamics. Such extension might shed more light onto the underlying boundary conformal theory, an issue indicated in ref. [16]. The solutions we have obtained using this method are connected by the duality relation $\lambda\rho(x) = \alpha - m(x)$, where $m(x)$ satisfies the BPS equation for the

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$\lambda' = 1/\lambda$ case. Owing to the $su(1,1)$ dynamical symmetry these solutions correspond to the quasi-classical CFT duals of the giant gravitons on AdS_5 ($\lambda < 1$) and on the sphere S^5 ($\lambda > 1$) [3, 10, 1, 2]. Further, the method has enabled us to solve the equations which govern the dynamics of quantum excitations around the uniform and various $x-$dependent backgrounds. These excitations described by the Hamiltonian in quadratic approximation represent quantum corrections to the semiclassical solutions. As an application of the results for quantum excitations on various $x-$dependent backgrounds, one could determine the correlation functions, the wave functionals of different states and the transition amplitudes among them. On the other hand, one could evaluate the same quantities using random matrix theory [40] (at least for $\lambda = 1, 1/2, 2$) and then compare the results.

We have found the finite form of the n-soliton solution in the duality-based matrix model, indicating the complete integrability of this model. Owing to the origin of the duality-based matrix model and to the properties of its semiclassical solutions, we interpret this matrix model as a model of interacting giant gravitons having cubic interaction. The BPS solutions for fields $\rho(x)$ and $m(x)$ (3.8) related by $\rho(x)m(x) = \tilde{c}$ admit an interesting singular limit $\rho(x)m(x) \to 0$; at the places where one field is different from zero the other field is vanishing. This "black/white" distribution is a characteristic of the two-dimensional droplet model of electron gas [41]. Finally, the singular limit of the n-soliton solution has motivated us to propose a realization of open-closed string duality. We have found the explicit mapping between the eigenstates of the collective-field Hamitonian and the eigenstates of the Hamiltonian describing quantum mechanics of nonrelativistic fermions. The proposal could be made more precise by expanding the relation (3.23) between the wave functional of the closed string excitations and the wave function of dual open string excitations around one of the soliton solutions found. We hope to address this issue in a future publication.

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